Alternating current

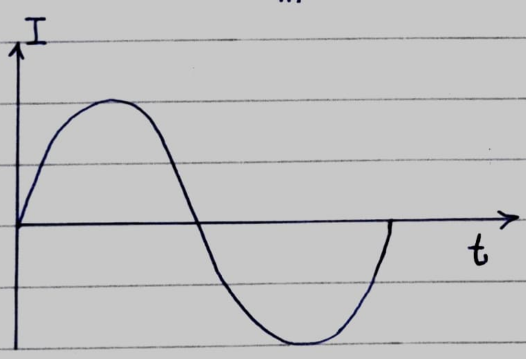
Unit ~ 2

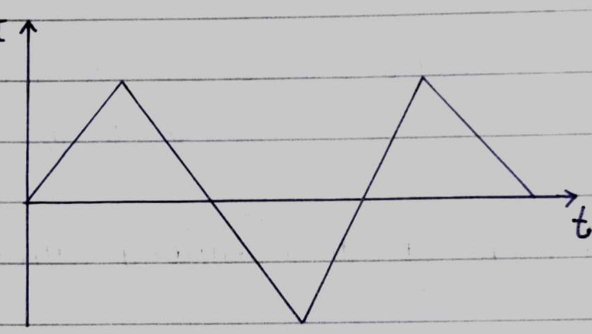
Condition required for current / voltage to be alternating:-

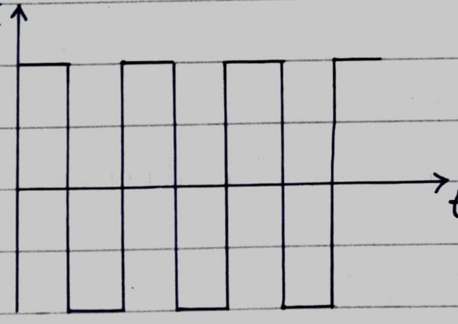
(i) Amplitude is constant.
(ii) Alternate half cycle is +ve and half -ve.
(iii) The alternete current continuously varis in magnitude and periodically reverses its direction.

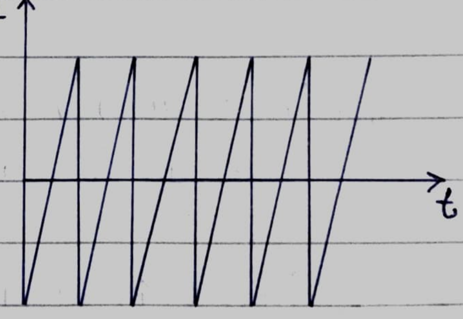
$$V = V_m \sin(\omega t + \phi) \quad \text{and} \quad V = V_m \cos(\omega t + \phi)$$

$$I = I_m \sin(\omega t + \phi) \quad \text{and} \quad I = I_m \cos(\omega t + \phi)$$









Phase and Phase difference:-

(a) Phase :-

$$I = I_m \sin(\omega t + \phi)$$

Initial phase = ϕ
Instantaneous phase = $\omega t + \phi$

Phase decides, both value and ~~dir~~ sign.

(b) Phase difference :-

$$V = V_m \sin(\omega t + \phi_1)$$
$$I = I_m \sin(\omega t + \phi_2)$$

Phase difference of I wrt V
$$\phi = \phi_2 - \phi_1$$

Phase difference of V wrt I
$$\phi = \phi_1 - \phi_2$$

Lagging and Leading concept :-

Leading :- jo pahle max par jayega.

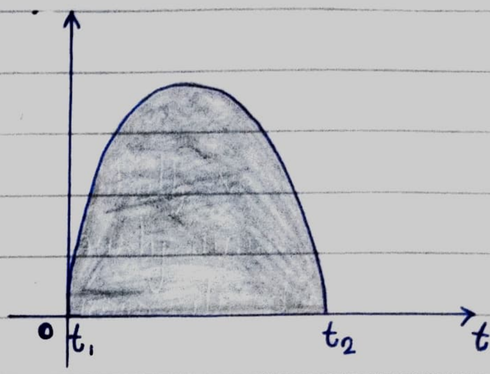
(1) $I = I_m \sin \omega t$ and $V = V_m \sin(\omega t + \phi)$
voltage is leading and current is lagging.

(2) $I = I_m \sin(\omega t + \phi)$ and $V = V_m \sin(\omega t)$
current is leading and voltage is lagging.

Average value and Root mean square value :-

Average value of any function :-

$$\langle f(t) \rangle = \frac{1}{T}\int_{t_1}^{t_2} f(t)\, dt$$



T = time periode

Root mean square value of any function :-

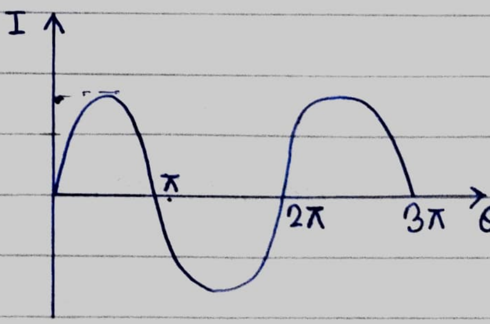
$$\text{Rms value of } f(t) = \sqrt{\frac{1}{T}\int_{t_1}^{t_2} (f(t))^2\, dt}$$

for complet cycle :-

$$\langle \sin\omega t \rangle = \langle \cos\omega t \rangle = \langle \sin 2\omega t \rangle = \langle \cos 2\omega t \rangle = 0$$

$$\langle \sin^2\omega t \rangle = \langle \cos^2\omega t \rangle = \frac{1}{2}$$

Ex :- 1. $I = I_m \sin\omega t = I_m \sin\theta$



$$I_{av} = \frac{1}{\pi}\int_0^{\pi} I_m \sin\theta\, d\theta$$

$$I_{av} = \frac{I_m}{\pi}\int_0^{\pi} \sin\theta\, d\theta$$

$$I_{av} = \frac{I_m}{\pi}(-\cos\theta)_0^{\pi} = \frac{I_m}{\pi}(1-(-1)) = \frac{2I_m}{\pi} = 0.637 I_m$$

avarage value of current = 0.637 × maximum value of current

R.m.s current $I = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} I_m^2 \sin^2\theta \, d\theta}$

$$I = \sqrt{\frac{I_m^2}{4\pi}\int_0^{2\pi}(1-\cos 2\theta)\, d\theta}$$

$$I = \sqrt{\frac{I_m^2}{4\pi}\left(\theta - \frac{\sin 2\theta}{2}\right)_0^{2\pi}}$$

$$I = \sqrt{\frac{I_m^2}{4\pi}(2\pi)}$$

$$I = \frac{I_m}{\sqrt{2}} = 0.707\, I_m$$

R.m.s value of current = 0.707 x maximum value of current

★ R.m.s. value is always greater then average value except in the case of a ractangular wave when both are equal.

Form factor :-

$$K_f = \frac{\text{rms value}}{\text{average value}}$$

for sinosoidal alternating voltage and current is

$$K_f = \frac{0.707\, E_m}{0.637\, E_m} = \frac{0.707\, I_m}{0.637\, I_m} = 1.11$$

Peak or crest or Amplitude factor :-

$$K_a = \frac{\text{maximum value}}{\text{rms value}} = \frac{I_m}{I_m/\sqrt{2}} = \frac{E_m}{E_m/\sqrt{2}} = 1.414$$

Ex:-2. Calculate the reading which will be a hot wire voltmeter if it is connected across the terminals of a generator whose voltage waveform is repersented by

$$V = 200 \sin \omega t + 100 \sin 3\omega t + 50 \sin 5\omega t$$

Solⁿ:- R.ms value $V = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} V^2\, d\theta}$ where $\theta = \omega t$

$$V^2 = \frac{1}{2\pi}\int_0^{2\pi}(200\sin\theta + 100\sin 3\theta + 50\sin 5\theta)^2\, d\theta$$

$$V^2 = \frac{1}{2\pi}\int_0^{2\pi}(200^2\sin^2\theta + 100^2\sin^2 3\theta + 50^2\sin^2 5\theta + 2\times 200\times 100$$

$$\sin\theta\sin 3\theta + 2\times 100\times 50\sin 3\theta\sin 5\theta + 2\times 50\times 200\sin 5\theta$$
$$\sin\theta)\, d\theta$$

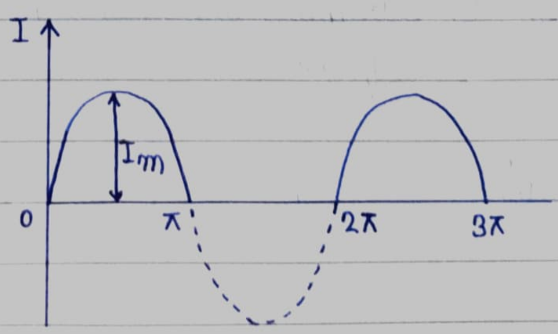
$$V^2 = \frac{1}{2\pi}\left(\frac{200^2}{2} + \frac{100^2}{2} + \frac{50^2}{2}\right)2\pi$$

$$V^2 = 26250$$

$$V = \sqrt{26250} = 162\,V$$

Rms value of H.W. Rectified alternating current :-

Rms current



$$I = \sqrt{\frac{1}{2\pi}\int_0^{\pi} I_m^2 \sin^2\theta\, d\theta}$$

$$I = \sqrt{\frac{I_m^2}{4\pi}\int_0^{\pi}(1 - \cos 2\theta)\, d\theta}$$

$$I = \sqrt{\frac{I_m^2}{4\pi}\left(\theta - \frac{\sin 2\theta}{2}\right)_0^{\pi}} = \sqrt{\frac{I_m^2}{4\pi} \times \pi}$$

$$I = \frac{I_m}{2} = 0.5\, I_m$$

Average value of H.W. Rectified alternating current :-

$$I_{av} = \int_0^{\pi} \frac{I_m \sin\theta \, d\theta}{2\pi}$$

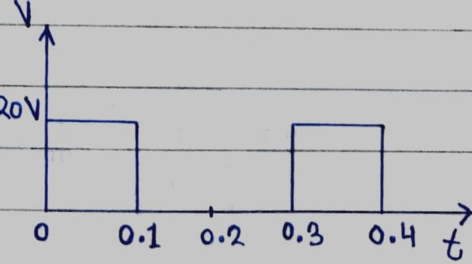
$$I_{av} = \frac{I_m}{2\pi}\left(-\cos\theta\right)_0^{\pi}$$

$$I_{av} = \frac{I_m}{\pi}$$

form factor $= \dfrac{I_m/2}{I_m/\pi} = \dfrac{\pi}{2} = 1.57$

peak factor $= \dfrac{I_m}{I_m/2} = 2$

Ex:- 3. Compute the average and effective values of the square voltage wave.



Sol^n :-

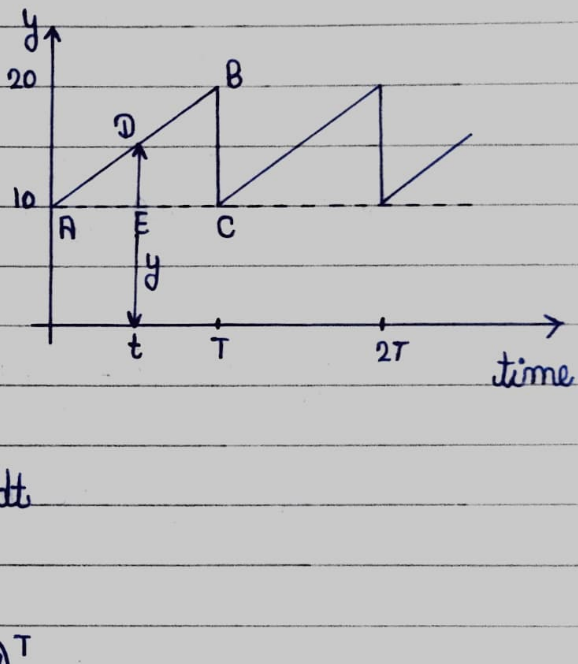
$$V_{av} = \frac{1}{T}\int_0^{T} v\,dt = \frac{1}{0.3}\int_0^{0.1} 20\,dt$$

$$V_{av} = \frac{1}{0.3}\left(20 \times 0.1\right) = 6.67\,V$$

$$V_{rms} = \sqrt{\frac{1}{0.3}\int_0^{0.1} 20^2\,dt}$$

$$V_{rms} = \sqrt{\frac{1}{0.3}(400 \times 0.1)} = \sqrt{133.3} = 11.5\,V$$

Ex:-4. What is the significance of the rms and average value of a wave? Determine the rms and average value of the waveform.



Sol^n:- The slope of the curve AB is $BC/AC = 10/T$.

It is seen that $DE/AE = BC/AC$
$= 10/T$

$$\frac{y-10}{t} = \frac{10}{T}$$

$$y = 10 + \frac{10t}{T}$$

$$Y_{av} = \frac{1}{T}\int_0^T \left(10 + \frac{10}{T}t\right) dt$$

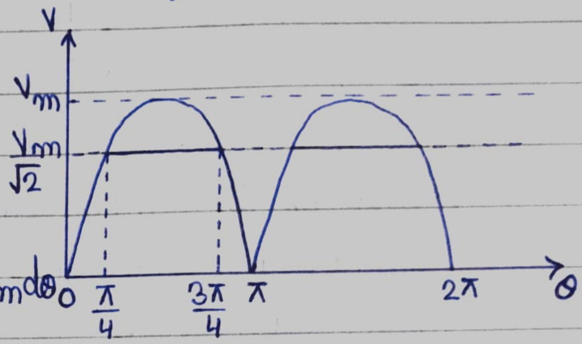
$$Y_{av} = \frac{1}{T}\left(10t + \frac{10}{2T}t^2\right)_0^T$$

$$Y_{av} = \frac{1}{T}\left(10T + 5T\right) = 15$$

$$Y_{rms} = \sqrt{\frac{1}{T}\int_0^T \left(10 + \frac{10t}{T}\right)^2 dt} = \sqrt{\frac{1}{T}\int_0^T \left(10^2 + \frac{10^2 t^2}{T^2} + \frac{200t}{T}\right) dt}$$

$$Y_{rms} = \sqrt{\frac{1}{T}\left(100 + \frac{100\,t^3}{3T^2} + \frac{200\,t^2}{2T}\right)_0^T} = \sqrt{\frac{700}{3}} = 15.2$$

Ex:- 5. A full wave rectified sinusoidal voltage is clipped at $1/\sqrt{2}$ of its maximum value. Calculate the average and rms values of such a voltage.



Solⁿ :-

$$V_{av} = \frac{1}{\pi}\left[\int_0^{\pi/4} v\,d\theta + \int_{\pi/4}^{3\pi/4} v\,d\theta + \int_{3\pi/4}^{\pi} v\,d\theta\right]$$

$$V_{av} = \frac{1}{\pi}\left[\int_0^{\pi/4} V_m \sin\theta\,d\theta + \int_{\pi/4}^{3\pi/4} 0.707 V_m\,d\theta + \int_{3\pi/4}^{\pi} V_m \sin\theta\,d\theta\right]$$

$$V_{av} = \frac{V_m}{\pi}\left[(-\cos\theta)_0^{\pi/4} + 0.707\,(\theta)_{\pi/4}^{3\pi/4} + (-\cos\theta)_{3\pi/4}^{\pi}\right]$$

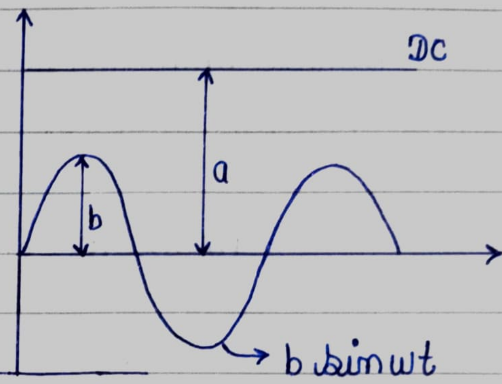
$$V_{av} = \frac{V_m}{\pi}\left(-\frac{1}{\sqrt{2}} + 1 + 0.707\left(\frac{3\pi}{4} - \frac{\pi}{4}\right) + (-1 + 1/\sqrt{2})\right)$$

$$V_{av} = \frac{V_m}{\pi}\left(-0.707 + 1 + 1.10 + 1 + 0.0707\right)$$

Ex:- 6. If a direct current of value a ampere is superimposed on an alternating current $I = b \sin \omega t$ flowing through a wire what is the effective value of the resulting current in the circuit?

Solⁿ:-

$$I = a + b \sin \omega t$$



$$I_{rms} = \sqrt{\langle I^2 \rangle}$$

$$I_{rms} = \sqrt{\langle (a + b \sin \omega t)^2 \rangle}$$

$$I_{rms} = \sqrt{\langle (a^2 + b^2 \sin^2 \omega t + 2ab \sin \omega t) \rangle}$$

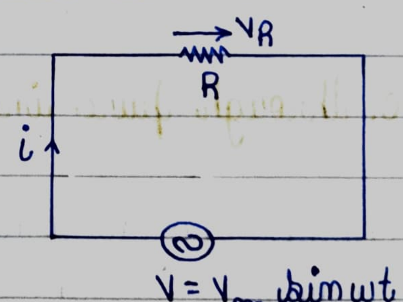
$$I_{rms} = \sqrt{a^2 + \frac{b^2}{2}}$$

A.C. through pure ohmic resistance alone :-

$$V = V_m \sin \omega t \quad \text{and} \quad I_m \sin \omega t = I$$

$$i_R = V_m \sin \omega t$$

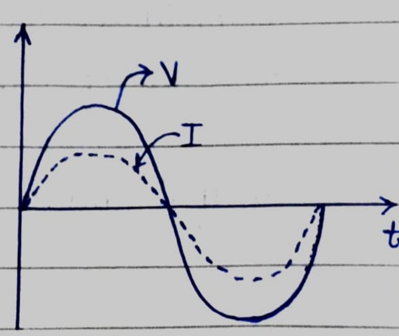
$$i = \frac{V_m}{R} \sin \omega t$$



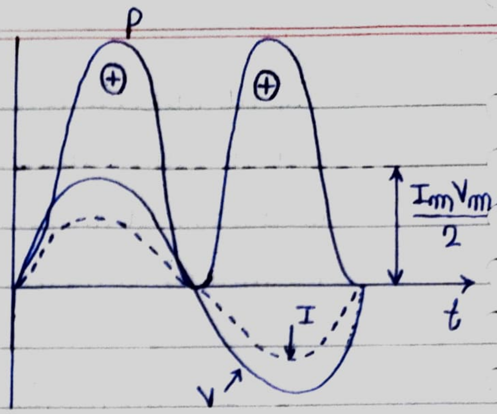
Instantaneous power $P = V(t) I(t)$

$$P = V_m I_m \sin^2 \omega t$$

$$P = \frac{V_m I_m}{2} - \frac{V_m I_m}{2} \cos 2\omega t$$



Power consists of a constant part $\frac{V_m I_m}{2}$ and a fluctuating part $\frac{V_m I_m}{2} \cos 2\omega t$ of frequency double that of voltage and current waves.



∵ for $\frac{V_m I_m}{2} \cos 2\omega t = 0$

Power of the whole cycle is

$$P = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}}$$

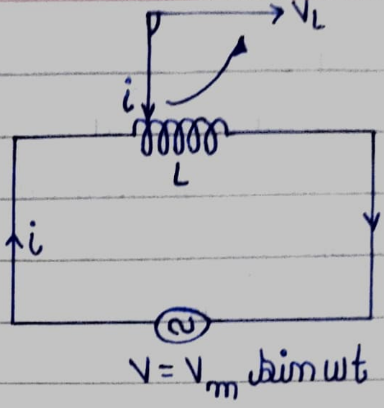
$$P = V_{rms} \times I_{rms}$$

In a purely resistive circuit, power is never 0. This is so because the instantaneous value of voltage and current are always either both positive or negative and hence the product is always +ve.

A.C. through pure inductance alone :-

Whenever an alternating voltage is applied to a purely inductive coil, a back EMF is produced due to self inductance of the coil. The back EMF opposes the rise or fall of the current through the coil. The applied voltage has to overcomes this self induced EMF, there is no ohmic voltage drop.

maximum energy stored $= E_m = \frac{1}{2} L I_m^2$



$$V = L \frac{di}{dt}$$

$$V_m \sin \omega t = L \frac{di}{dt}$$

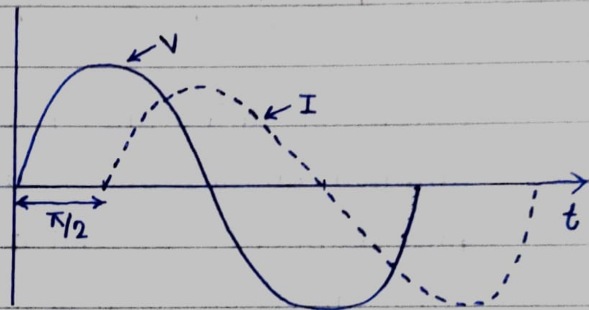
$$di = \frac{V_m}{L} \sin \omega t \, dt$$

$$i = \frac{V_m}{L} \int \sin \omega t \, dt$$

$$i = \frac{V_m}{L\omega} (-\cos \omega t)$$

$$i = \frac{V_m}{X_L} \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$I_m = \frac{V_m}{X_L}$$



The current lags behind the applied voltage. The phase difference between the two is $\pi/2$ with voltage leading.

Inductive Reactance $(X_L) = \omega L = 2\pi f L$ in ohm.

Instantaneous power $= V_m I_m \sin \omega t \sin(\omega t - \pi/2)$

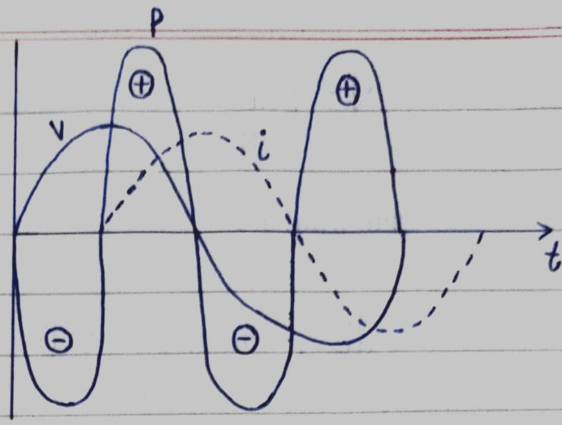
$$P = -V_m I_m \sin \omega t \cos \omega t$$

$$P = -\frac{V_m I_m}{2} \sin 2\omega t$$

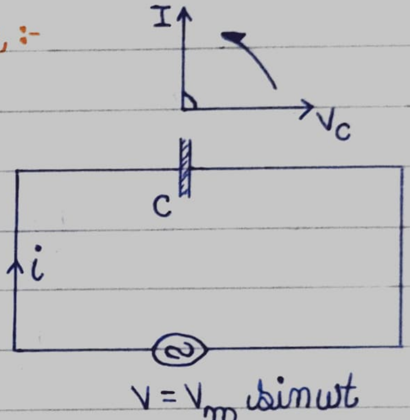
The average demand of power from the supply for a complete cycle is 0.

The max. value of the instantcetaneous power is $\frac{V_m I_m}{2}$.

Here again it is seen that power wave is a sine wave of frequency double that of the voltage and current waves.



A.C. through pure capacitance alone :-



When an alternating voltage is applied to the plates of a capacitor the capacitor is charged first in one direction and then in the opposite direction.

$$q = CV = CV_m \sin\omega t$$

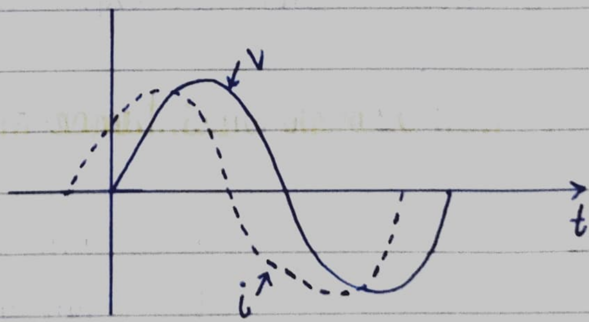
$$i = \frac{dq}{dt} = \frac{d}{dt}(CV_m \sin\omega t)$$

$$i = (CV_m \cos\omega t)\omega$$

$$i = \frac{V_m}{1/\omega C}\cos\omega t$$

$$i = \frac{V_m}{X_C}\cos\omega t$$

$$i = \frac{V_m}{X_C}\sin\left(\omega t + \frac{\pi}{2}\right)$$



$$I_m = \frac{V_m}{X_C}$$

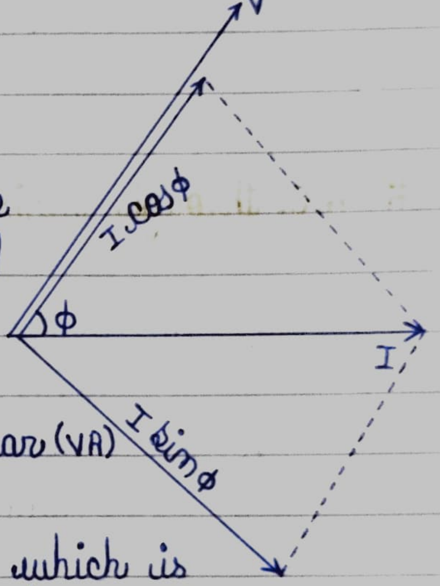
Capacitive reactance $(X_C) = \frac{1}{\omega C}$

Impedance vector has numerical value of $\sqrt{R^2+X_L^2}$.
Its phase angle with be the reference axis is $\phi = \tan^{-1}\frac{X_L}{R}$
It may also be expressed in the polar form as $Z = Z\angle\phi$.

→ Power factor :-

$$\text{Power factor} = \cos\phi = \frac{R}{Z}$$

Active, Reactive and Apparent power :-



(i) **Apparent power (S)** :- It is given by the product of rms values of applied voltage and circuit current.

$$S = VI = (IZ)I = I^2Z \text{ volt ampear (VA)}$$

(ii) **Active power (P/W)** :- It is the power which is actually dissipated in the circuit resistance.

$$P = I^2R = VI\cos\phi \text{ watt}$$

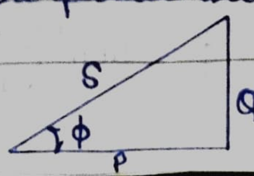
(iii) **Reactive power (Q)** :- It is the power ~~which is actually dissipated in the circuit resistance~~ developed in the inductive reactance of the circuit.

$$Q = I^2X_L = I^2Z\sin\phi = VI\sin\phi \text{ volt amperes reactive (VAR)}$$

These three powers are shown in the power triangle

$$S^2 = P^2 + Q^2$$

$$S = \sqrt{P^2+Q^2}$$



Q factor of a coil :- Reciprocal of power factor is called the Q-factor of a coil.
It is also known as quality factor of a coil.

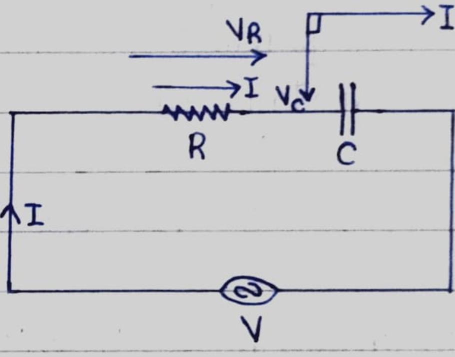
$$Q \text{ factor} = \frac{1}{\cos\phi} = \frac{Z}{R}$$

$$Q \text{ factor} = 2\pi \frac{\text{maximum energy stored}}{\text{energy dissipated per cycle}}$$

A.C. through resistance and capacitance :-

$V_R = IR$ = drop across R in phase with I.

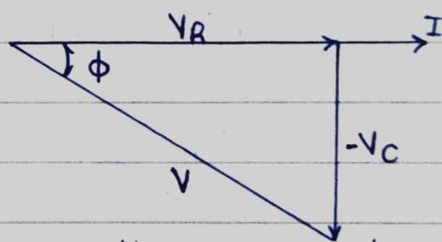
$V_C = IX_C$ = drop across capacitor lagging I by $\pi/2$.



$$V = \sqrt{V_R^2 + (-V_C)^2}$$

$$V = \sqrt{(IR)^2 + (-IX_C)^2}$$

$$V = I\sqrt{R^2 + X_C^2}$$



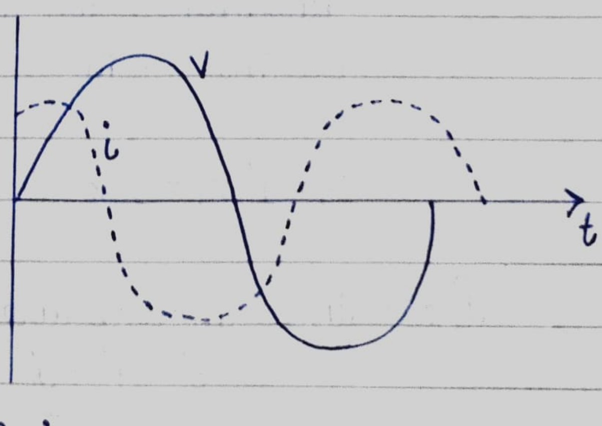
voltage triangle

$$I = \frac{V}{\sqrt{R^2 + X_C^2}} = \frac{V}{Z}$$

$$Z = \sqrt{R^2 + X_C^2}$$

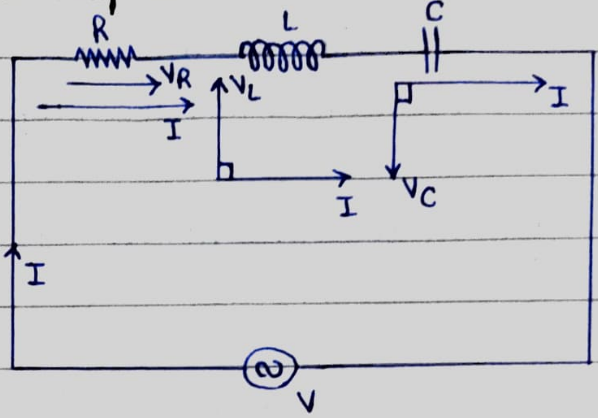
$$\tan\phi = \frac{-X_C}{R}$$

$$\phi = \tan^{-1}\left(\frac{-X_C}{R}\right)$$



$$Z = R - jX_C$$

Resistance, Inductance and capacitance in series :-



$V_R = IR$ = voltage drop across R.

$V_L = IX_L$ = voltage drop across L.

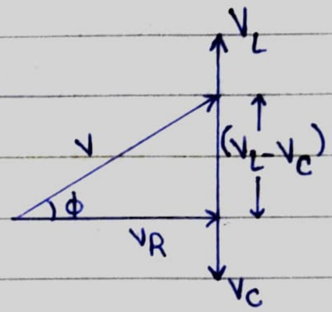
$V_C = IX_C$ = voltage drop across C.

$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$V = \sqrt{I^2(R^2 + (X_L - X_C)^2}$$

$$V = I\sqrt{R^2 + (X_L - X_C)^2}$$

$$V = IZ$$



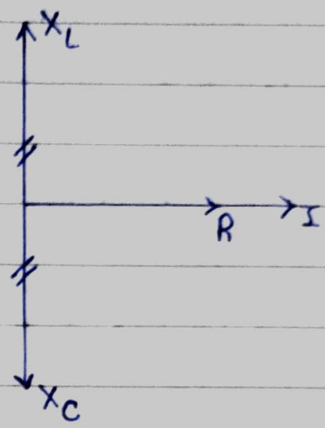
$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\tan\phi = \frac{(X_L - X_C)}{R} \Rightarrow \phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

power factor $= \cos\phi = \frac{R}{Z}$

$$Z = R + j(X_L - X_C)$$

Resonance :- A circuit is said to be resonant when the natural frequency of circuit is equal to frequency of the applied voltage.



In series Resonance $X_L - X_C = 0$

$$X_L = X_C$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2}$$

$$Z = R$$

Impedence will be minimum.

$$I_{max} = \frac{V}{Z} = \frac{V}{R}$$

phase $= \phi = 0°$

power factor $= \cos\phi = \cos 0° = 1$

? Calculation of Resonant frequency :- The frequency at which the net reactance of the series circuit is 0 is called the resonant frequency f_0.

$$V_L = V_C$$

$$I X_L = I X_C \Rightarrow X_L = X_C$$

$$\omega L = \frac{1}{\omega C}$$

$$\omega^2 = \frac{1}{LC}$$

$$\omega = \frac{1}{\sqrt{LC}}$$

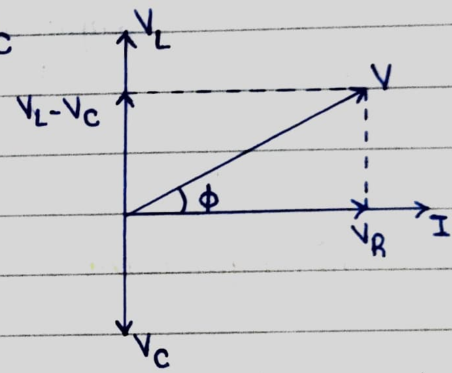
$$2\pi f_0 = \frac{1}{\sqrt{LC}} \Rightarrow f_0 = \frac{1}{2\pi\sqrt{LC}}$$

A series resonant circuit is sometimes called acceptor circuit and the series resonance is often referred to as voltage resonance.

Incidentally, it may be noted that if X_L and X_C are shown at any frequency f, that the value of the resonant frequency of such a circuit can be found by the relation $f_0 = f\sqrt{\frac{X_C}{X_L}}$.

Inductive R-L-C circuit :- $V_L > V_C$

$X_L > X_C$



$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$
$$V = \sqrt{I^2 (R^2 + (X_L - X_C)^2)}$$
$$V = I\sqrt{R^2 + (X_L - X_C)^2}$$

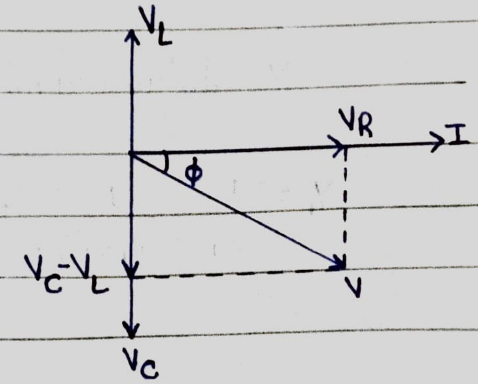
$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

phase $= \phi > 0$, power factor $= \cos\phi > 0$

V is leading I by phase ϕ.

Capacitive R-L-C circuit :- $V_L < V_C$

$X_L < X_C$



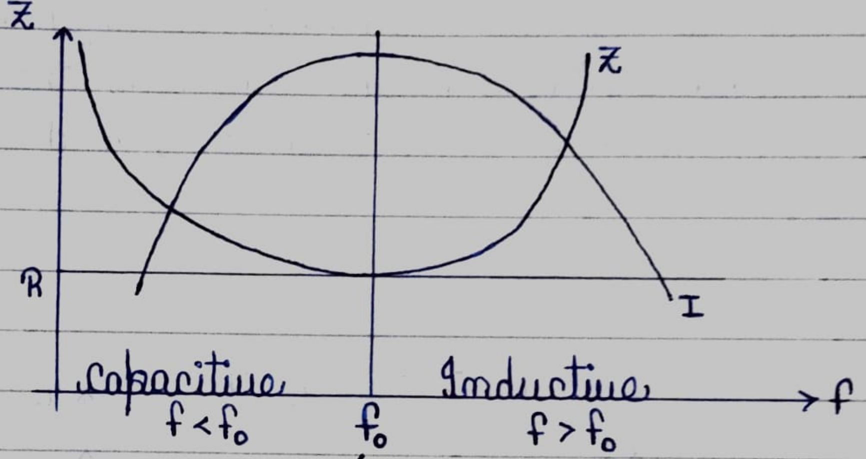
$$V = IZ$$

$$Z = \sqrt{R^2 + (X_C - X_L)^2}$$

phase $= \phi < 0$, power factor $= \cos\phi > 0$

$\tan\phi < 0$

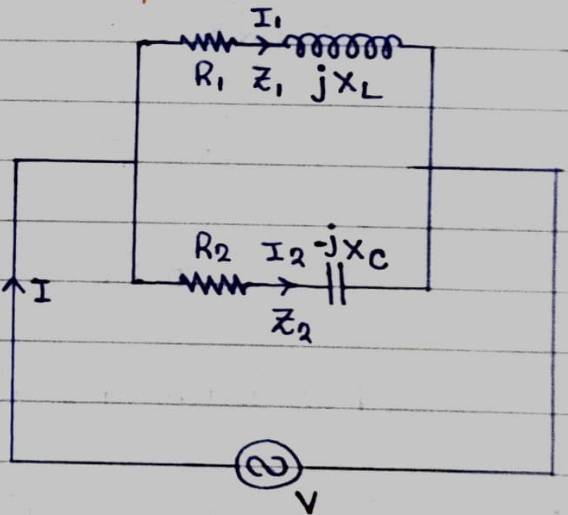
voltage laging by phase ϕ.



at resonance

Algebra

solving parallel circuits by complex or phasor method :-



$I_1 = \frac{V}{Z_1}$ and $I_2 = \frac{V}{Z_2}$

$I = I_1 + I_2 = \frac{V}{Z_1} + \frac{V}{Z_2}$

$I = V\left(\frac{1}{Z_1} + \frac{1}{Z_2}\right) = V(Y_1 + Y_2)$

$I = VY$

$Y =$ total admittance $= Y_1 + Y_2$

$Y_1 = \frac{1}{Z_1} = \frac{1}{R_1 + jX_L} = \frac{R_1 - jX_L}{R_1^2 + X_L^2}$

$$Y_1 = \frac{R_1}{R_1^2 + X_L^2} - j\frac{X_L}{R_1^2 + X_L^2}$$

$$Y_1 = g_1 + j b_1$$

conductance of upper branch $= g_1 = \dfrac{R_1}{R_1^2 + X_L^2}$

susceptance of upper branch $= b_1 = -\dfrac{X_L}{R_1^2 + X_L^2}$

$$Y_2 = \frac{1}{Z_2} = \frac{1}{R_2 - jX_C} = \frac{R_2 + jX_C}{R_2^2 + X_C^2}$$

$$Y_2 = \frac{R_2}{R_2^2 + X_C^2} + j\frac{X_C}{R_2^2 + X_C^2}$$

$$Y_2 = g_2 + j b_2$$

$g_2 = \dfrac{R_2}{R_2^2 + X_C^2}$ and $b_2 = \dfrac{X_C}{R_2^2 + X_C^2}$

Total admittance $= Y = Y_1 + Y_2 = (g_1 - jb_1) + (g_2 + jb_2)$

$Y = (g_1 + g_2) - j(b_1 - b_2)$ and $\phi = \tan^{-1}\dfrac{(b_1 - b_2)}{(g_1 + g_2)}$

$$Y = G - jB$$

$Y = \sqrt{G^2 + B^2}$ and $\tan\phi = \dfrac{B}{G}$

$I = VY \angle\phi$; $I_1 = VY_1$ and $I_2 = VY_2$

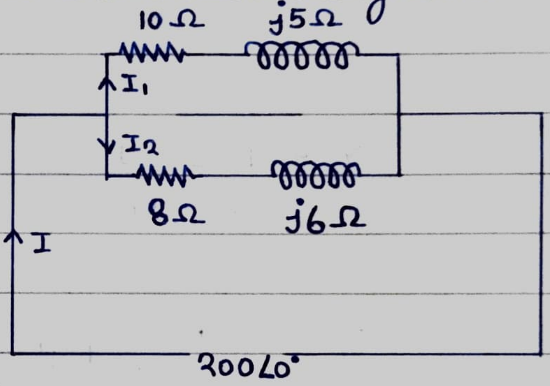
If $V = V\angle\alpha$ and $Y = Y\angle\beta$ then $I = VY$

$$I = V\angle\alpha \times Y\angle\beta$$

$$I = VY\ \angle\alpha+\beta$$

Ex:- Two impedances gives by $Z_1 = 10+j5$ and $Z_2 = 8+j6$ are joined in parallel and connected across a voltage of $V = 200+j0$. Calculate the circuit current, its phase and the branch current. Draw the vector diagram.

Solⁿ:-



$$Y_1 = \frac{1}{Z_1} = \frac{1}{10+j5} = \frac{10-j5}{(10)^2+(5)^2}$$

$$Y_1 = \frac{10}{125} - j\frac{5}{125}$$

$Y_1 = 0.08 - j0.04$ simens

$$Y_2 = \frac{1}{Z_2} = \frac{1}{8+j6} = \frac{8-j6}{8^2+6^2} = \frac{8}{100} - j\frac{6}{100}$$

$Y_2 = 0.08 - j0.06$ siemens

$Y = Y_1 + Y_2 = 0.16 - j0.1$ siemens

$$I = VY = (200+j0)(0.16-j0.1)$$

$$I = 32 - j20$$

$$I = 37.74\ \angle -32^\circ$$

$$I_1 = VY_1 = (200+j0)(0.08-j0.04)$$

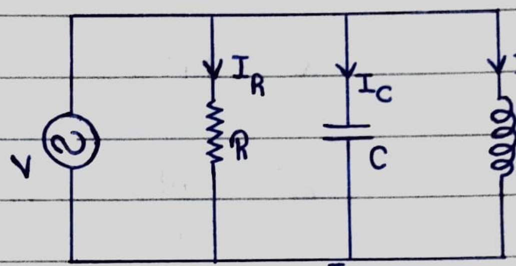
$$I_1 = 16 - j8$$

$$I_1 = 17.88\ \angle -26.56^\circ$$

$$I_2 = VY_2 = (200 + j0)(0.08 - j0.06)$$
$$= 16 - j12$$
$$I_2 = 20\angle -36.86^\circ$$

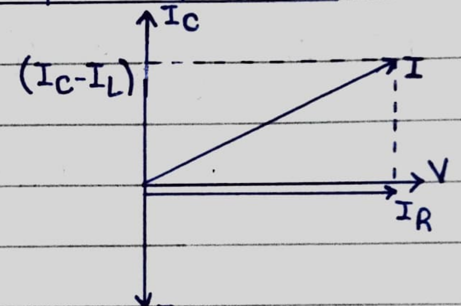
$$P.f. = \cos\phi = \cos 32^\circ = 0.848$$

Parallel R-L-C circuit :-



$$I = \sqrt{I_R^2 + (I_C - I_L)^2}$$

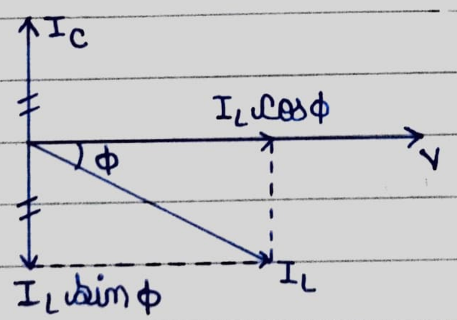
$$I = \sqrt{\frac{V^2}{R^2} + \left(\frac{V}{X_C} - \frac{V}{X_L}\right)^2}$$

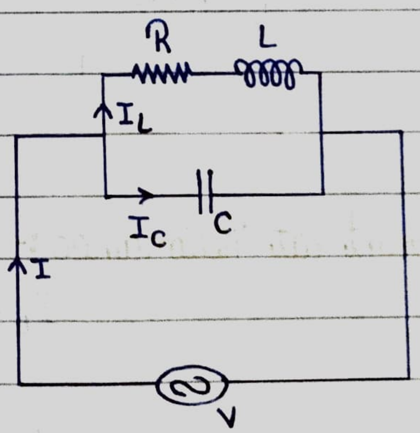


$$I = V\sqrt{\frac{1}{R^2} + \left(\frac{1}{X_C} - \frac{1}{X_L}\right)^2}$$

$$Z = \sqrt{\frac{1}{R^2} + \left(\frac{1}{X_C} - \frac{1}{X_L}\right)^2}$$

Resonance in Parallel circuits :-





net reactive or wattless component = 0

$$I_C - I_L \sin\phi = 0$$

$$I_L \sin\phi = I_C$$

$$I_L = \frac{V}{Z}, \quad \sin\phi = \frac{X_L}{Z} \quad \text{and} \quad I_C = \frac{V}{X_C}$$

$$\frac{V}{Z} \times \frac{X_L}{Z} = \frac{V}{X_C}$$

$$Z^2 = X_L X_C$$

$$Z^2 = \frac{\omega L}{\omega C} \Rightarrow Z^2 = \frac{L}{C} \Rightarrow Z = \sqrt{\frac{L}{C}}$$

$$R^2 + X_L^2 = \frac{L}{C} \Rightarrow X_L^2 = \frac{L}{C} - R^2$$

$$2\pi f_0 = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$\boxed{f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}}$$

if $R << L$

$$\boxed{f_0 = \frac{1}{2\pi\sqrt{LC}}}$$

7 Current at Resonance :-

$$I = I_L \cos\phi = \frac{V}{Z} \times \frac{R}{Z}$$

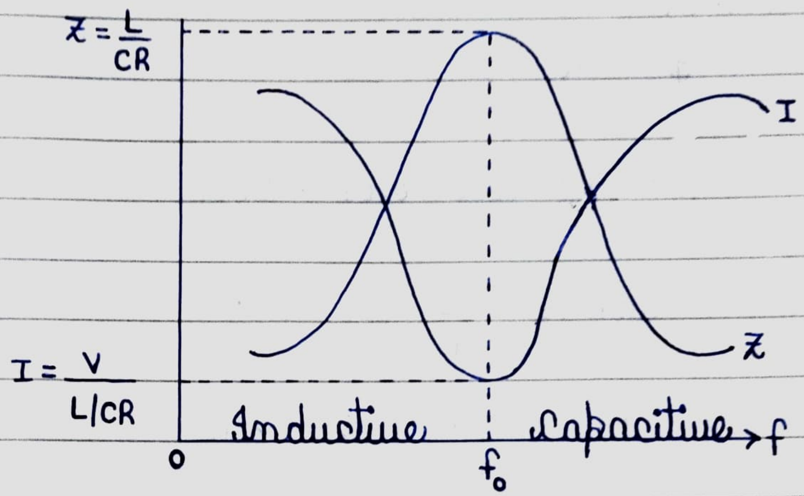
$$I = \frac{VR}{Z^2} \qquad \because Z^2 = L/C$$

$$I = \frac{VR}{L/C}$$

$$\boxed{I = \frac{V}{L/RC}}$$

The denominator L/CR is known as the equivalent or dynamic impedance of the parallel circuit at resonance.

"Z max and current is min."



Q - factor of a Parallel circuit :-

$$\text{Q-factor} = \frac{I_C}{I} = \frac{V/X_C}{V/(L/RC)} = \frac{V/(1/\omega C)}{VRC/L}$$

$$= \frac{V\omega C \times L}{VRC}$$

$$= \frac{\omega L}{R}$$

$$\text{Q-factor} = \frac{2\pi f_0 L}{R}$$

f_0 at $R \ll L$ $\quad f_0 = \dfrac{1}{2\pi\sqrt{LC}}$

$$\boxed{\text{Q-factor} = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

Ex:-1. A resistance R, an inductance $L = 0.01H$ and a capacitance C are connected in series. When a voltage $V = 400 \cos(3000t - 10^\circ)$ V is applied to the series combination, the current flowing is $10\sqrt{2}\cos(3000t - 55^\circ)$ A. find R and c.

Sol^n:-

$$\phi = -10^\circ - (-55^\circ) = 45^\circ$$

$$X_L = \omega L = 3000 \times 0.01 = 30\Omega$$

$$\tan 45^\circ = \frac{X}{R} \Rightarrow X = R$$

$$Z = \frac{V_m}{I_m} = \frac{400}{10\sqrt{2}} = 28.28\,\Omega$$

$$Z^2 = R^2 + X^2 \Rightarrow Z^2 = R^2 + R^2$$

$$Z^2 = 2R^2$$

$$Z = \sqrt{2}R$$

$$R = \frac{28.28}{\sqrt{2}} = 20\,\Omega$$

$$X = X_L - X_C \Rightarrow 20 = 30 - X_C$$

$$X_C = 10\,\Omega$$

$$X_C = \frac{1}{\omega C}$$

$$C = \frac{1}{X_C \omega} = \frac{1}{10 \times 3000} = \frac{1}{3 \times 10^4}$$

$$C = 0.33 \times 10^{-4} = 33\mu F$$

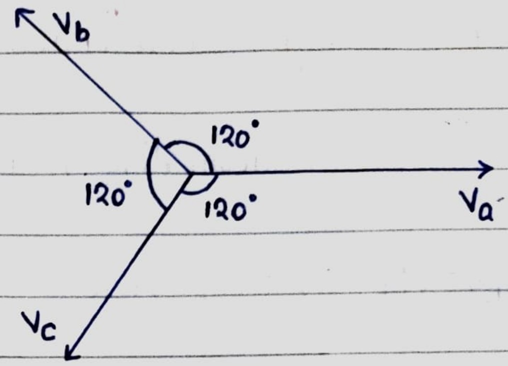
Three-phase balanced circuit :-

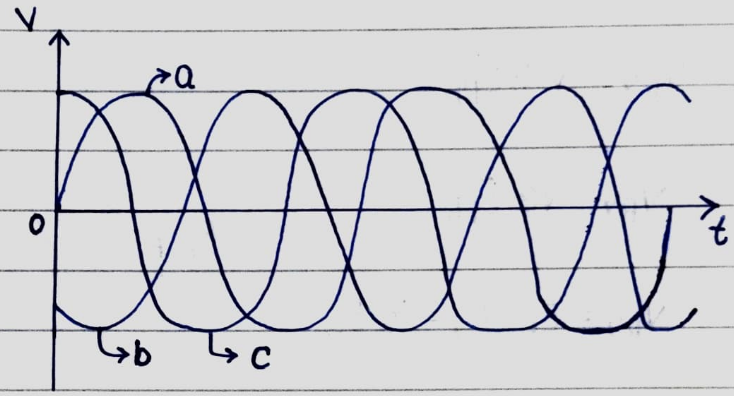
$V_a = V_m \sin \omega t$

$V_b = V_m \sin(\omega t - 120^\circ)$

$V_c = V_m \sin(\omega t - 240^\circ)$

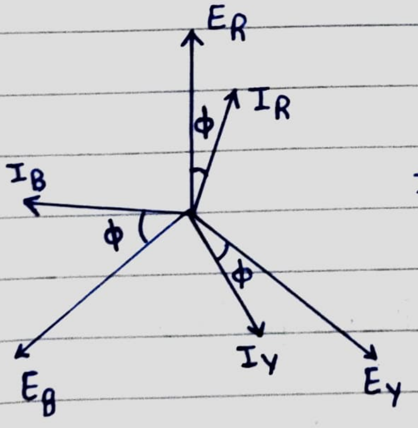
or $V_m \sin(\omega t + 120^\circ)$

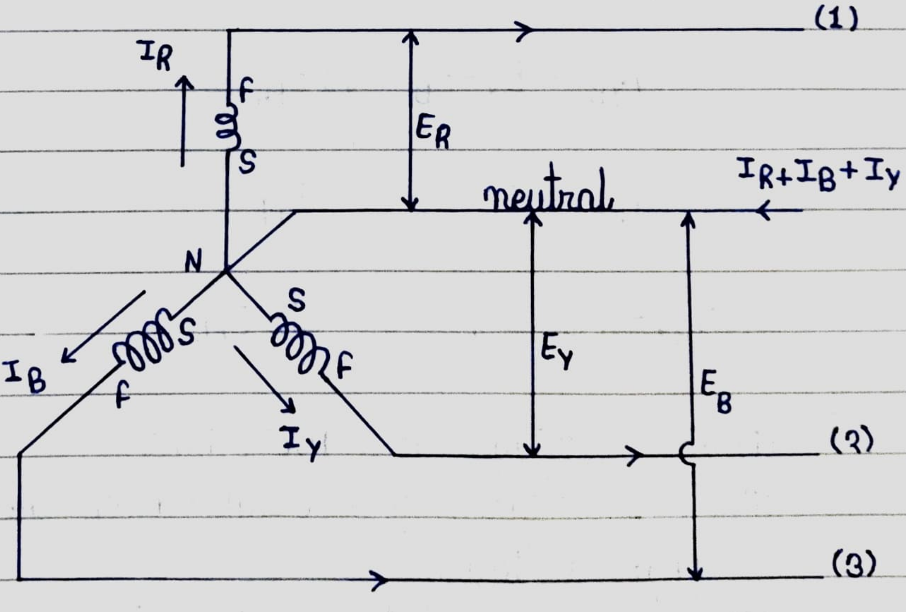




Star or Wye (Y) Connection :-

$I_R + I_B + I_Y = 0$





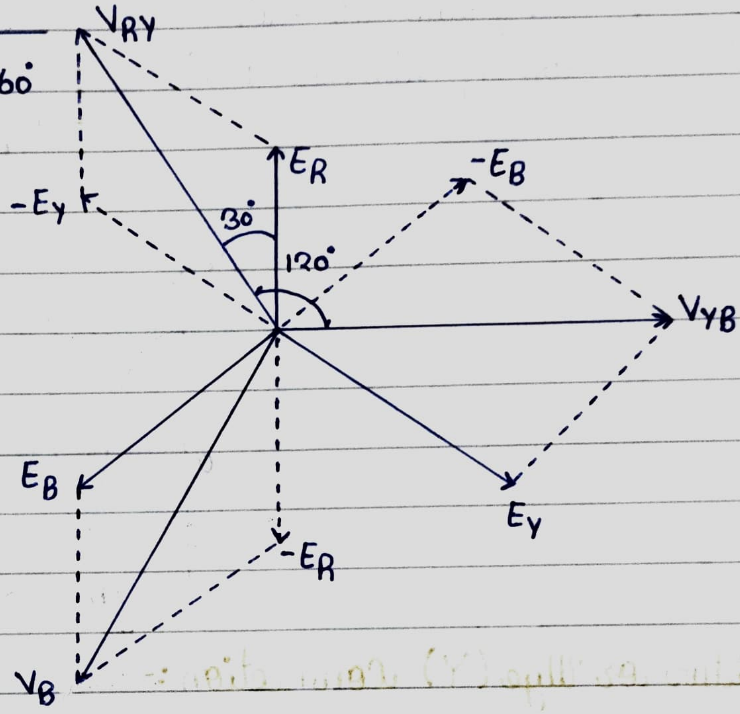
In balanced system has been assumed

$E_R = E_Y = E_B = E_{Ph}$ (phase EMF)

Line voltages and Phase voltages :-

The p.d between line 1 and 2 is $V_{RY} = E_R - E_Y$

$$V_{RY} = \sqrt{E_R^2 + E_Y^2 + 2E_RE_Y \cos 60^\circ}$$



$$V_{RY} = \sqrt{E_{Ph}^2 + E_{Ph}^2 + 2E_{Ph}^2 \times \frac{1}{2}}$$

$$V_{RY} = \sqrt{3E_{Ph}^2}$$

$$V_{RY} = \sqrt{3}\, E_{Ph}$$

$$V_{RY} = E_R - E_Y = \sqrt{3}\, E_{Ph}$$

Similarly -

$$V_{YB} = E_Y - E_B = \sqrt{3}\, E_{Ph}$$

$$V_{BR} = E_B - E_R = \sqrt{3}\, E_{Ph}$$

$$V_{RY} = V_{YB} = V_{BR} = V_L = \sqrt{3}\, E_{Ph}$$

★

(i) Line voltages are 120° apart.

(ii) Line voltages are 30° ahead of their respective phase voltages.

(iii) The angle between the line currents and the corresponding line voltage is $(30 + \phi)$ with current lagging.

Line currents and Phase currents :-

Current in line 1 = I_R , Current in line 2 = I_Y
current in line 3 = I_B

line current $I_L = I_{Ph}$

Power :- The total active or true power in the circuit is the sum of the three phase powers.

total active power = $P_1 + P_2 + P_3$

$$P = V_{Ph} I_{Ph} \cos\phi + V_{Ph} I_{Ph} \cos\phi + V_{Ph} I_{Ph} \cos\phi$$

$$P = 3 V_{Ph} I_{Ph} \cos\phi$$

$$V_{Ph} = \frac{V_L}{\sqrt{3}} \quad \text{and} \quad I_{Ph} = I_L$$

$$P = 3 \frac{V_L}{\sqrt{3}} I_L \cos\phi$$

$$P = \sqrt{3} V_L I_L \cos\phi$$

ϕ is the angle between phase voltage and phase current.

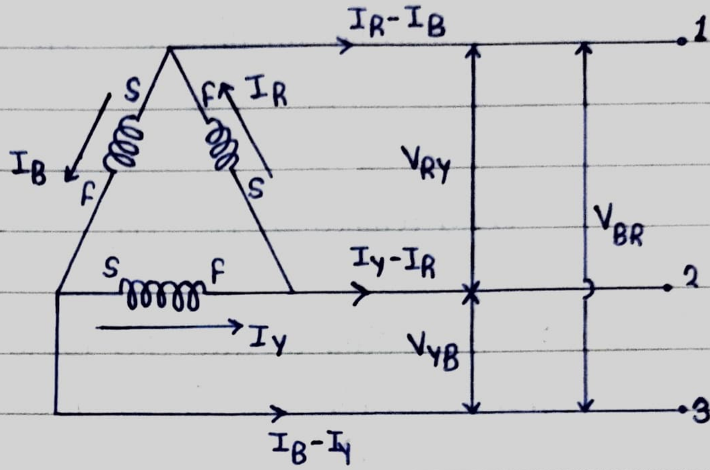
Total Reactive power $Q = \sqrt{3} V_L I_L \sin\phi$

Reactive power of a coil is taken as positive and that of a capacitor as negative.

power absorbed by each phase = $I_{Ph}^2 R_{Ph}$

Total apparent power $S = \sqrt{P^2 + Q^2} = \sqrt{3}\, V_L I_L$

Delta (Δ) or Mesh connection :-



i) Line voltages and Phase voltages :-

voltage between line 1 and 2 as V_{RY}.
voltage between line 2 and 3 as V_{YB}.
voltage between line 3 and 1 as V_{BR}.

V_{RY} lead V_{YB} by 120° and V_{YB} lead V_{BR} by 120°.

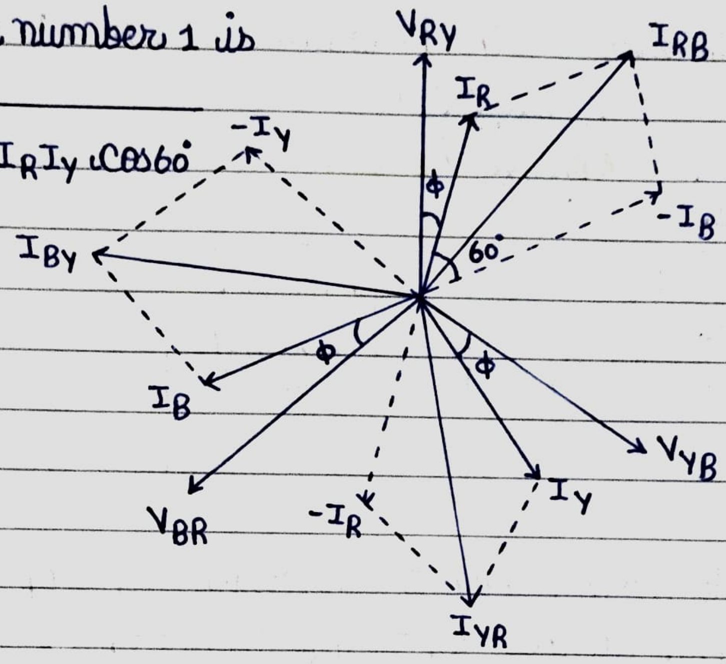
$$V_{RY} = V_{YB} = V_{BR} = V_L$$

$$V_L = V_{Ph}$$

ii) Line current and Phase current :-

Current in line 1 is $I_R - I_B$.
Current in line 2 is $I_Y - I_R$.
Current in line 3 is $I_B - I_R$.

Current in line number 1 is



$$I_{RY} = \sqrt{I_R^2 + I_Y^2 + 2 I_R I_Y \cos 60°}$$

$$I_R = I_Y = I_{Ph}$$

$$I_{RY} = \sqrt{3 I_{Ph}^2}$$

$$I_{RY} = \sqrt{3}\, I_{Ph}$$

similarly

$$I_{BY} = \sqrt{3}\, I_{Ph}$$

$$I_{RB} = \sqrt{3}\, I_{Ph}$$

$$I_L = \sqrt{3}\, I_{Ph}$$

☆ (i) Line currents are 120° apart.
(ii) Line currents are 30° behind the respective phase currents.
(iii) The angle between the line currents and the corresponding line voltages is $(30° + \phi)$ with the current lagging.

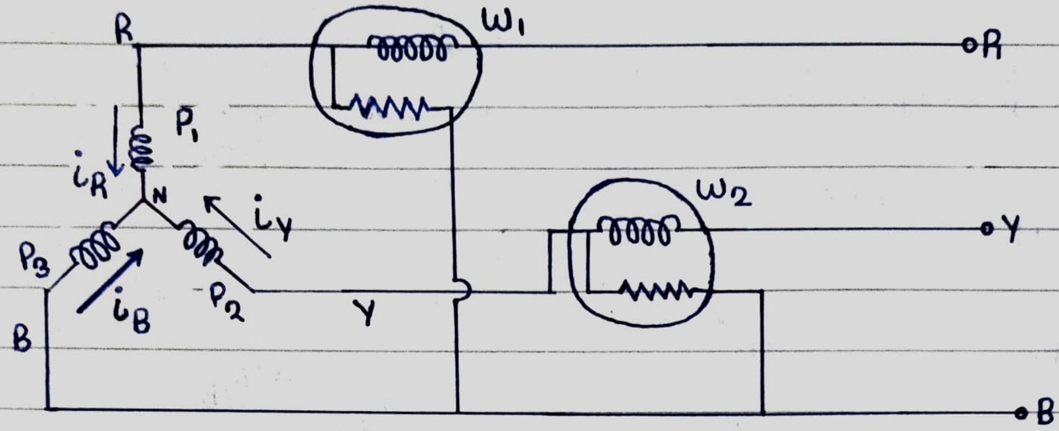
Power :-

$$P = 3 V_{Ph} I_{Ph} \cos\phi$$

$$P = 3 V_L \frac{I_L}{\sqrt{3}} \cos\phi \qquad \because V_{Ph} = V_L, \quad I_{Ph} = \frac{I_L}{\sqrt{3}}$$

$$P = \sqrt{3}\, V_L I_L \cos\phi$$

Two Wattmeter method - Balanced load :-



Instanteneous current through $W_1 = i_R$

p.d. across $W_1 = V_{RB} = E_R - E_B$

power read by $W_1 = i_R (E_R - E_B)$

Instantaneous current through $W_2 = i_Y$

p.d. across $W_2 = V_{YB} = E_Y - E_B$

power read by $W_2 = i_Y (E_Y - E_B)$

$$W_1 + W_2 = i_R(E_R - E_B) + i_Y(E_Y - E_B)$$
$$W_1 + W_2 = i_R E_R - i_R E_B + i_Y E_Y - i_Y E_B$$
$$W_1 + W_2 = i_R E_R + i_Y E_Y - E_B(i_R + i_Y)$$

$$W_1 + W_2 = i_R E_R + i_Y E_Y + E_B i_B$$

$\because i_R + i_Y + i_B = 0$

$i_R + i_Y = -i_B$

$$W_1 + W_2 = P_1 + P_2 + P_3$$

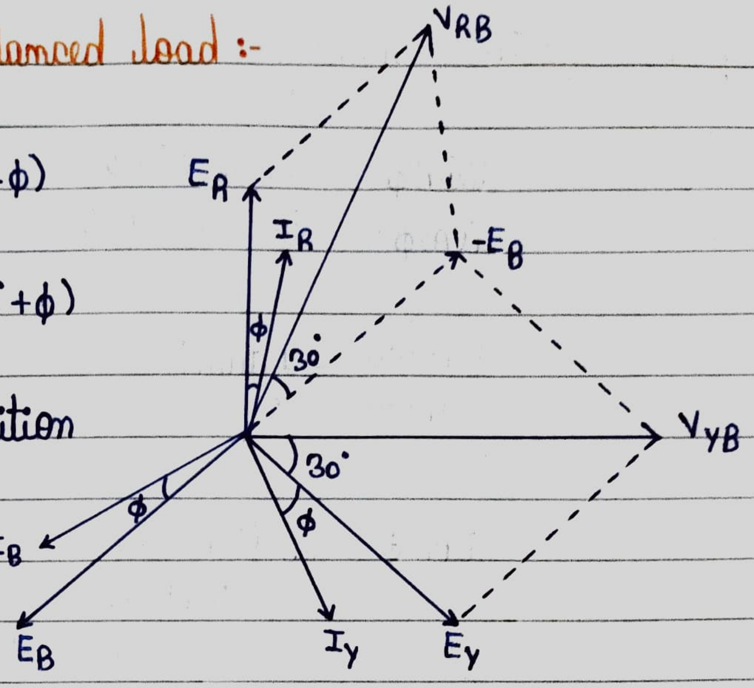
$W_1 + W_2 =$ total power absorbed

same formula for delta load connection.

Power Factor - Balanced load :-

$$W_1 = I_R V_{RB} \cos(30^\circ - \phi)$$

$$W_2 = I_Y V_{YB} \cos(30^\circ + \phi)$$



in balanced condition

$V_{RB} = V_{YB} = V_L$

and $I_R = I_Y = I_L$

$$W_1 = I_L V_L \cos(30^\circ - \phi)$$
$$W_2 = I_L V_L \cos(30^\circ + \phi)$$

$$W_1 + W_2 = I_L V_L \cos(30^\circ - \phi) + I_L V_L \cos(30^\circ + \phi)$$

$$W_1 + W_2 = I_L V_L [\cos 30^\circ \cos\phi + \sin 30^\circ \sin\phi + \cos 30^\circ \cos\phi - \sin 30^\circ \sin\phi]$$

$$W_1 + W_2 = I_L V_L (2 \cos 30^\circ \cos\phi)$$
$$W_1 + W_2 = \sqrt{3} I_L V_L \cos\phi$$

$$W_1 - W_2 = I_L V_L \cos(30^\circ - \phi) - I_L V_L \cos(30^\circ + \phi)$$

$$W_1 - W_2 = I_L V_L [\cos 30^\circ \cos\phi + \sin 30^\circ \sin\phi - \cos 30^\circ \cos\phi + \sin 30^\circ \sin\phi]$$

$$W_1 - W_2 = I_L V_L (2 \sin 30^\circ \sin\phi)$$

$$W_1 - W_2 = I_L V_L \sin\phi$$

Power factor lagging :-

$$\frac{W_1 - W_2}{W_1 + W_2} = \frac{I_L V_L \sin\phi}{\sqrt{3} I_L V_L \cos\phi}$$

$$\frac{W_1 - W_2}{W_1 + W_2} = \frac{1}{\sqrt{3}} \tan\phi$$

$$\tan\phi = \sqrt{3}\,\frac{(W_1 - W_2)}{W_1 + W_2}$$

Power factor leading :-

$$\tan\phi = -\sqrt{3}\,\frac{(W_1 - W_2)}{W_1 + W_2}$$

Ex:- 1. A star connected alternator supplies a delta connected load. The impedance of the load branch is $(8+j6)$. The line voltage is 230V. find
considerd delta connection
(a) Current in load branch.

Solⁿ :-

$$V_{ph} = V_L = 230V$$

$$Z_{ph} = \sqrt{8^2 + 6^2} = \sqrt{100} = 10$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{230}{10}$$

$$I_{ph} = 23\ A$$

(b) Power consumed by the load.

Sol^n:-

$$I_L = \sqrt{3} I_{Ph} = \sqrt{3} \times 23 = 39.8 \text{ A}$$

$$P = I_L V_L \sqrt{3} \cos\phi \qquad \because \cos\phi = \frac{R}{Z}$$

$$P = \sqrt{3} \times 39.8 \times 230 \times 0.8$$

$$P = 12684 \text{ W}$$

(c) power factor of the load.

Sol^n:-

$$P.f = \cos\phi = \frac{R}{Z_{Ph}} = \frac{8}{10} = 0.8 \text{ (lag)}$$

(d) Reactive power of the load.

Sol^n:-

$$Q = \sqrt{3} V_L I_L \sin\phi$$

$$Q = \sqrt{3} \times 230 \times 39.8 \times 0.6 \qquad \because \sin\phi = \frac{X_L}{Z}$$

$$Q = 9513 = 9.513 \text{ KVA}$$

Ex:-2. A balanced delta connected load, consisting of there coils, draws $10\sqrt{3}$ A at 0.5 power factor from 100V, 3-phase supply. If the coil are re-connected in star across the same supply, find the line current and total power consumed.

Sol^n:- Delta connection

$$V_{Ph} = V_L = 100\text{V}$$

$$I_L = 10\sqrt{3} \text{ A}$$

$$I_{ph} = \frac{10\sqrt{3}}{\sqrt{3}} = 10A$$

$$Z_{ph} = \frac{V_{ph}}{I_{ph}} = \frac{100}{10} = 10\,\Omega$$

$\cos\phi = 0.5$

$\sin\phi = \sqrt{1-(0.5)^2}$

$\sin\phi = 0.866$

$R_{ph} = Z_{ph}\cos\phi$

$R_{ph} = 10 \times 0.5$

$R_{ph} = 5\,\Omega$

$X_{ph} = Z_{ph}\sin\phi$

$X_{ph} = 10 \times 0.866$

$X_{ph} = 8.66\,\Omega$

total power consumed $= \sqrt{3} V_L I_L \cos\phi$

$= \sqrt{3} \times 100 \times 10\sqrt{3} \times 0.5$

$= 1500W$

star connection

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{100}{\sqrt{3}}\,V$$

$Z_{ph} = 10\,\Omega$

$I_{ph} = I_L = 10\sqrt{3}\,A$

total power absorbed $= \sqrt{3} \times 100 \times 10\sqrt{3} \times 0.5$

$= 500W$